REPORT DOCUMENTATION PAGE

AFRL-SR-AR-TR-03-

0283

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE April 28, 2003	3. REPORT TYPE AND DATES COVERED Final Progress Report – 9/1/01 – 12/31/02

4. TITLE AND SUBTITLE
Analysis and Distributed Control Of A Formation Of Intelligent Satellites

5. FUNDING NUMBERS
F49620-01-1-0518

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8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Air Force Office of Scientific Research
4015 Wilson Blvd Room 713
Arlington, VA 22203

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release;
distribution unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

Perturbation based methods for determining initial conditions and shaping satellite formations to accommodate nonlinearity of the differential gravitational acceleration as well as eccentricity of the reference orbit have been developed. Control laws for formation maintenance as well as reconfiguration, valid for large formations and eccentric reference orbits have been developed via filter based LQR designs and period matching. An analytical solution to the relative motion problem has been obtained by using a unit-sphere description of the motion. This approach converts the problem of formation control into an attitude control problem. A sub-optimal strategy for reconfiguring a formation using impulsive thrust has been developed that does not require any off-line optimization. The results of this approach have been compared with those obtained from numerical optimization. An analytical technique has also been developed for designing formations for large and high eccentricity relative orbits

14. SUBJECT TERMS

15. NUMBER OF PAGES
1 – 7 includes SF298

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAR (same as report)

20030602 131

ANALYSIS AND DISTRIBUTED CONTROL OF A FORMATION OF INTELLIGENT SATELLITES
Final Report

F49620-01-1-0518

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Objectives

Our research objectives involve the following:

- Formations in resonant orbits
- Formations in highly eccentric orbits
- Formation reconfiguration
- Reference orbit determination
- Formation control
- Differential drag and solar radiation effects

Status of Effort

Perturbation based methods for determining initial conditions and shaping satellite formations to accommodate nonlinearity of the differential gravitational acceleration as well as eccentricity of the reference orbit have been developed. Control laws for formation maintenance as well as reconfiguration, valid for large formations and eccentric reference orbits have been developed via filter based LQR designs and period matching. An analytical solution to the relative motion problem has been obtained by using a unit-sphere description of the motion. This approach converts the problem of formation control into an attitude control problem. A sub-optimal strategy for reconfiguring a formation using impulsive thrust has been developed that does not require any off-line optimization. The results of this approach have been compared with those obtained from numerical optimization. An analytical technique has also been developed for designing formations for large and high eccentricity relative orbits

Accomplishments

PERTURBATION SOLUTION TO NONLINEAR RELATIVE MOTION EQUATIONS

Hill-Clohessy-Wiltshire (HCW) equations describe the relative motion of a satellite with respect to another in a circular reference orbit. Initial conditions that generate periodic solutions to these equations have to be corrected in order to obtain bounded solutions in the presence of nonlinearity of the differential gravitational acceleration model and eccentricity of the reference orbit. We have established corrections to the initial conditions due to quadratic terms in the differential gravitational acceleration for circular reference orbits using a perturbation approach. These corrections have been related to the period matching constraint required for bounded relative motion. Next, the solution to the linear problem including the effect of eccentricity is determined. The two solutions obtained are combined to produce an asymptotic solution for the quadratic, eccentricity problem. The effects of nonlinearity and eccentricity on the relative motion of individual satellites have been investigated. The following figures illustrate our results for two satellites with different phase angles.

SUB-OPTIMAL RECONFIGURATION OF SPACECRAFT FORMATIONS IN EARTH ORBITS

To achieve the desired objectives of a formation-flying mission, it is often necessary for the formation to reconfigure itself. In this work, we analyze the formation reconfiguration problem for formations in orbits given by the HCW periodic solutions. The desired formations are characterized by the orbital elemental differences. Gauss's variational equations are used to compute impulses that establish the desired orbital elemental differences. An analytical, sub-

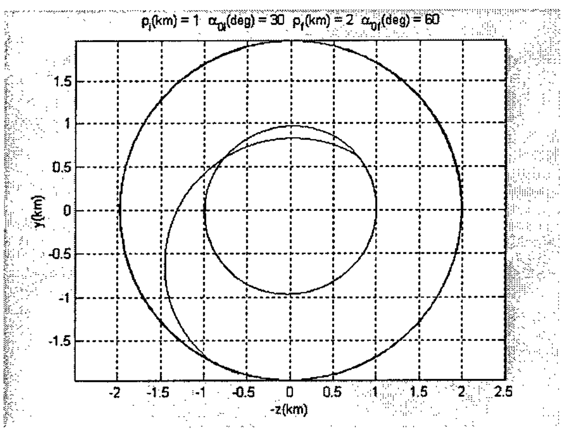

Fig. 1 Reconfiguration Example

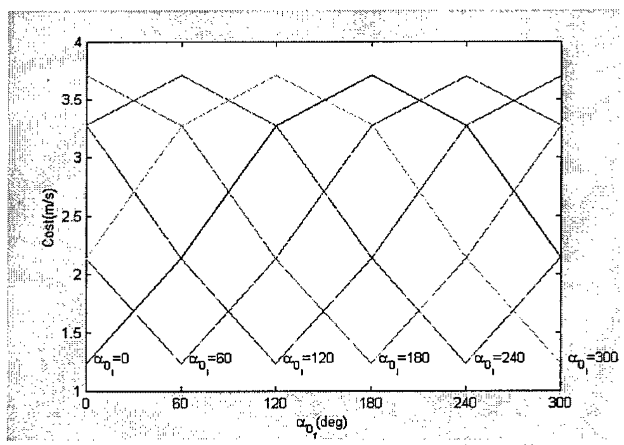

Fig. 2 Reconfiguration Cost for Individual Satellites

optimal solution is proposed that can be easily implemented without requiring any online optimization. The analytical solution also assigns to each spacecraft in the initial configuration a unique slot in the final configuration, such that the overall fuel consumption is minimized. The solution is also extended to accommodate the J_2 perturbation. The cost incurred with the analytical solution is found to be close to that incurred by the optimal solution obtained by a numerical optimization procedure. Figure 1 shows a reconfiguration from a 1 km circular projection relative orbit to a 2 km circular projection relative orbit. The chief satellite is at the origin of the coordinate system. The initial phase angle or the phase angle in the initial orbit is 30° and the phase angle in the final orbit is 60°. This reconfiguration assumes that there is a single thruster on requires only two impulses.

Figure 2 shows a plot of the ΔV required for different satellites in an initial configuration to transfer to different locations in a final configuration. It can be seen that the individual minimum for each satellite occurs at a unique value of the final slot. The overall fuel consumption is minimized by assigning to each satellite a slot that corresponds to its minimum ΔV.

ANALYTICAL SOLUTIONS FOR THE RELATIVE MOTION OF SATELLITES VALID FOR HIGH ECCENTRICITY ORBITS

The positions of the Chief and Deputy are projected onto a unit sphere by normalizing their positions using their respective distances from the center of the Earth. This results in analytical expressions for the so-called "sub-satellite" points that are functions of the angles only (right ascension Ω, inclination i, and argument of latitude θ).

$$\begin{Bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{Bmatrix} = [\mathbf{C}_0\mathbf{C}_1 - \mathbf{I}]\begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$$
$$\mathbf{C}_0 \equiv \mathbf{C}_0(\Omega_0, i_0, \theta_0) \tag{1}$$
$$\mathbf{C}_1 \equiv \mathbf{C}_1(\Omega_1, i_1, \theta_1)$$

C_0 and C_1 are the direction cosine matrices of the Chief and Deputy, respectively, which are functions of their respective angles. The analytical solution to the along-track position is

$$\begin{aligned} \Delta y &= c^2(i_0/2)c^2(i_1/2)s(\Delta\theta + \Delta\Omega) + s^2(i_0/2)s^2(i_1/2)s(\Delta\theta - \Delta\Omega) \\ &\quad - s^2(i_0/2)c^2(i_1/2)s(2\theta_0 + \Delta\theta + \Delta\Omega) - c^2(i_0/2)s^2(i_1/2)s(2\theta_0 + \Delta\theta - \Delta\Omega) \\ &\quad + 1/2s(i_0)s(i_1)[s(\Delta\theta) + s(2\theta_0 + \Delta\theta)] \end{aligned} \tag{2}$$

The true relative position and velocity vectors are then obtained by scaling, using the radial distances of the Chief and Deputy, r_C and r_D, respectively.

To study the effect of J_2, we may use either osculating elements or mean elements. For large eccentricity orbits, series expansions in the powers of eccentricity prove annoying since 1) the number of terms required in the series expansion is not known, and 2) convergence of the series is not guaranteed. In the new method, instead of stepping through time, we step through the true anomaly of the Chief. This does away with the necessity of solving Kepler's equation for the Chief, and we only need to perform the calculations for the Deputy. Usually, wherever mean elements are considered, we essentially take into account only the secular growth due to the J_2 perturbation. For the semi-major axis a, eccentricity and inclination, there is no secular growth whereas in Ω, ω, and M we use the mean rates. If we also take into account the short-period variations, then the accuracy of the method is improved. This leads to corrections in Ω, i, and θ, as well as r that are obtained from Kozai's work, which can be incorporated in the algorithm.

As an example, we consider an eccentricity of 0.8182 and a relative orbit size of 10-20 km, established using a node difference. The results obtained from the analytical solution are compared with those obtained from numerical integration with and without the short period corrections. The first set of figures in Fig. 3 shows the errors between the analytical mean element solution and the numerical solution. The second set of figures show the errors obtained after the short period corrections are incorporated. The addition of the short period corrections leads to a more accurate result.

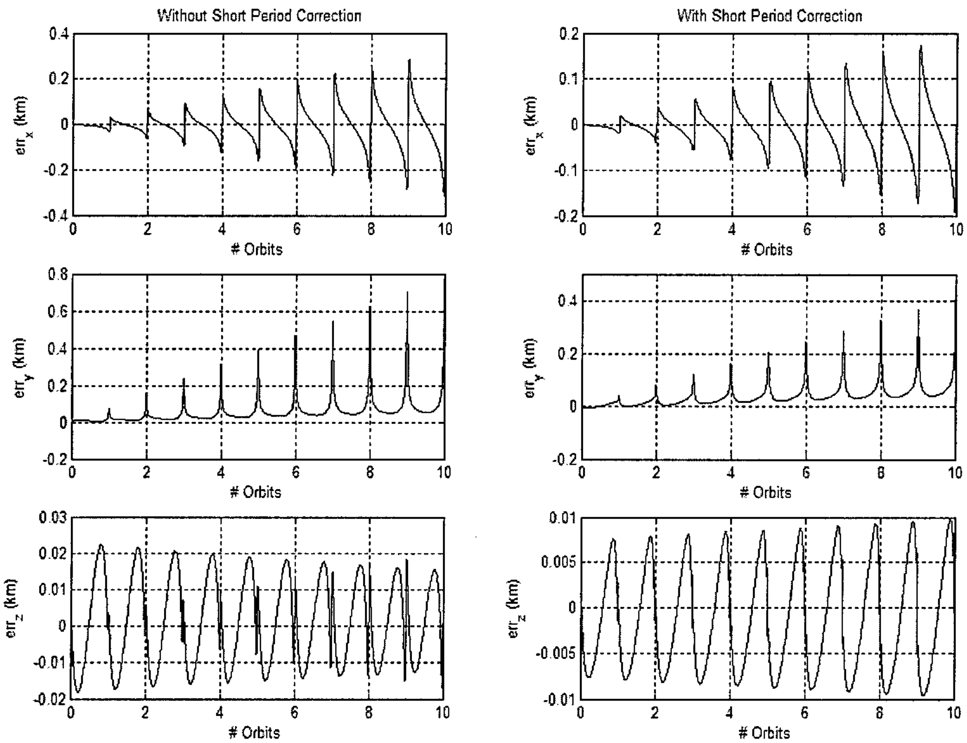

Figure 3 Errors between the analytical solutions and numerical integration for a high eccentricity Orbit.

LINEAR AND NONLINEAR CONTROLLERS FOR FORMATION FLYING

In this work, we analyze different control schemes for formation flying satellites. The objective is to devise control strategies that can stabilize large formations optimally. Three types of controllers have been studied in this work: (i) Lyapunov stabilized controller, (ii) LQR controllers, and (iii) Period matching controllers. The Lyapunov controller offers global stability and zero steady state tracking errors but the associated control cost is very high. The LQR controllers guarantee only local stability but offer significant cost benefits. The period matching controllers exploit the existence of control-free natural solutions and force the dynamics to the nearest period matched trajectory. The period matching control law is globally stable and results in the desired relative orbits at a very low control cost.

CLASSIFICATION OF RELATIVE MOTION ORBITS

In this paper presented at the 2002 AAS Space Flight Mechanics Conference we developed classification of the types of relative motion orbits according to the number of constraints. This classification applies to orbits for which the primary perturbation results from differential gravity. With each class of orbits the fuel or Δv needed to counter the differential gravitational perturbation effects was provided. Differences in the momenta or action variables between the deputy and chief are what cause drift between the satellites. The semi-major axis, eccentricity and inclination define the generalized momenta so the constraints are defined by differences in these three quantities. In Class 1 relative motion orbits there are three constraints, the semi-major axis, eccentricity and inclination of the deputy and chief are equal. An example of Class 1 is the leader-follower with out-of-plane motion caused by a right ascension difference. In Class 1 no fuel s needed to counter the differential gravitational perturbations. In Class 2 orbits there are two constraints allowing one degree of freedom in the three momenta variables. One constraint is the change in semi-major axis needed to negate the in-track drift resulting from changes in the eccentricity and inclination. The 2nd constraint negates the out-of-plane drift or perigee drift. The J_2 invariant orbits developed in the first Techsat21 grant fall into this class. Class 3 orbits have only one constraint and that is the constraint on the semi-major axis to negate the secular in-plane drift due to changes in the eccentricity and inclination. Almost all bounded relative motion orbits can occur in this class, but need some thrusting to maintain.

Personnel Supported
K. T. Alfriend (Professor), S. R. Vadali (Professor), D. W. Gim (PhD student), and S. Vaddi (PhD student).

Interactions
1. K. T. Alfriend presented a seminar at UCSD in May.
2. K. T. Alfriend gave a keynote lecture at the 2002 Nonlinear Problems in Aviation and Aerospace Conference in Melbourne, FL.
3. K. T. Alfriend gave an invited lecture on "Dynamics and Control of Formation Flying Satellites" at the Astrodynamics Workshop which was one of the workshops under the Geometric Mechanics & Symmetry Workshop sponsored by the University of Warwick in Great Britain.
4. S. R. Vadali attended the DCSSS Conference in Cambridge, England and presented the paper "An Analytical Solution for Relative Motion of Satellites."
5. Vadali, S. R. and Vaddi S., "Large-Angle Kinematics and Control of Satellite Relative Motion," AIAA/AAS Astrodynamics Conference, Monterey, CA, August 2002.
6.S. S. Vaddi, Vadali S. R., and Alfriend, K.T., "Formation Flying: Accommodating Nonlinearity and Eccentricity Perturbations," AAS/AIAA Space Flight Mechanics Conference, San Antonio Texas, Paper AAS 02-184, January 27-30 2002.
7 Alfriend, K.T., Gim D-W., and Vadali S. R., "The Characterization of Formation Flying Satellite Relative Motion Orbits," AAS/AIAA Space Flight Mechanics Conference, San Antonio Texas, Paper AAS 02-143, January 27-30 2002.
8. S. S. Vaddi and Vadali S. R., "Linear and Nonlinear Control Laws for Formation Flying," AAS/AIAA Space Flight Mechanics Conference, Puerto Rico, Paper AAS 03-109, February 10-13 2003.
9.Vadali, S. R. and Vaddi S., "Analytical Solution to the Satellite Relative Motion Problem Using Mean Orbital Elements," AIAA/AAS Astrodynamics Conference, Monterey, CA, August 2002.
10. S. S. Vaddi, K. T. Alfriend and S. R. Vadali, "Sub-Optimal Formation Reconfiguration Scheme Using Impulsive Control", Submitted to the 2003 AAS/AIAA Astrodynamics Specialist Conference at Big Sky, Montana, Aug 3-7 2003.

Transitions

Concepts developed with the STM and Impulse control will be used for TechSat21. Point of contact: Dr. Craig McLaughlin.
We are in the process of transitioning our research in formation flying to the PowerSail project of AFRL. Point of contact: Dr. Greg Spanjers, Dr. Brian Engberg, and Dr. T. A. Lovell.

Honors/Awards

During grant:
1. D. W. Gim and K. T. Alfriend received the Best Paper Award for the AAS Flight Mechanics Conference, 2001.
2. S. Vaddi was the runner-up in the Graduate Student Paper Competition.
3. S. R. Vadali received the TEES Fellow, E. D. Brockett Professor, Stewart and Stevenson Professor awards.

Prior to grant:
Alfriend, K.T. Fellow AIAA (1988), Fellow AAS (1985), Fellow International Academy of Astronautics (1998), AIAA Mechanics and Control of Flight Award (1998), AAS Dirk Brouwer Award (1989)

Publications

1. S. S. Vaddi, Vadali S. R., and Alfriend, K.T., "Formation Flying: Accommodating Nonlinearity and Eccentricity Perturbations," *Journal of Guidance, Control, and Dynamics,* March-April 2003, pp. 214-223.
2. Vadali, S.R., Vaddi, S.S. and Alfriend, K.T., "An Intelligent Control Concept for Formation Flying Satellite Constellations," *International Journal of Nonlinear and Robust Control,* dedicated to Formation Flying, 2002; 12:97-115.
3. Vadali, S.R., Vaddi S. S., Naik, K., and Alfriend, K. T., "Control of Satellite Formations in General Periodic Relative Orbits," to be submitted for publication in the *Journal of Guidance, Control, and Dynamics.*